and so on, where $m=\frac{1}{2}(n-1)$. We can now deduce that

$$E(\overset{*}{\gamma}/\gamma)^s=n^s\int_0^\infty c(x,n)\left(\sum\frac{f_s(x)}{\gamma^s}\right)\times\left\{\frac{1+\sqrt{(1+4x/3n\gamma)}}{4x}\right\}^s dx,\quad (s=1,2\ldots). \quad (33)$$

In particular, expanding the integrand to include the first two dominant terms, we have

$$E(\overset{*}{\gamma}/\gamma)^s\sim n^s\int_0^\infty\frac{e^{-x}x^{\frac{1}{2}(n-3)}}{\left(\frac{n-3}{2}\right)!(4x)^s}\left(1+\frac{f_1(x)}{\gamma}+\ldots\right)\times\left(2+\frac{2x}{3n\gamma}+\ldots\right)^s dx=\frac{n^s}{(n-3)(n-5)\ldots(n-2s-1)}\times\left\{1-\frac{s(s+1)}{3n\gamma}+\ldots\right\}. \quad (34)$$

This is exactly the same as the result for the maximum likelihood moment $E\ (\hat{\gamma}/\gamma)^s$ given in Shenton and Bowman (1969), so that the higher moments of Thom's statistic $\overset{*}{\gamma}$ are asymptotically equivalent to the corresponding moments of the maximum likelihood estimator of γ.

Similarly, by using the independence property given in section 6 in conjunction with (3b), an expression can be found for the asymptotic form of $E\ (\overset{*}{\beta}/\beta)^s$.

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